Econometrica Supplementary Material

SUPPLEMENT TO “TASK TRADE BETWEEN SIMILAR COUNTRIES”

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BY GENE M. GROSSMAN AND ESTEBAN ROSSI-HANSBERG

IN THIS SUPPLEMENTAL MATERIAL, we prove Lemmas 1, 2, and 3.

LEMMA 1: *If $w > w^*$, either* (i) $\tilde{\pi}(i) > 0$ *or* $\tilde{\pi}(i) < 0$ *for all i or* (ii) $J > 0$ *and* $\tilde{\pi}(i) > 0$ *for $i < J$ while* $\tilde{\pi}(i) < 0$ *for $i > J$.*

PROOF: Without loss of generality, assume $w > 1$ (given $w^* = 1$). The aggregate cost of performing task i in East minus the aggregate cost of performing it in West is proportional to

$$\begin{aligned}\Lambda(i; nx, n^*x^*, w) &\equiv \tilde{\pi}(i; nx, n^*x^*, w) A(nx + n^*x^*) \\ &= (wnx - n^*x^*) - \beta t(i)(nx - wn^*x^*).\end{aligned}$$

First assume that $n^*x^* \geq nx$. Then $nx - wn^*x^* < 0$, which implies $\min_i \Lambda(i; nx, n^*x^*, w) = \Lambda(0; nx, n^*x^*, w)$ since $t'(i) > 0$ for all i. Then, since $\beta t(0) > 1$,

$$\begin{aligned}\Lambda(0; nx, n^*x^*, w) &> wnx - n^*x^* - nx + wn^*x^* \\ &= (w - 1)(nx + n^*x^*) > 0.\end{aligned}$$

So all tasks have higher aggregate cost in East; that is, $\tilde{\pi}(i) > 0$ for all i and $J = 1$.

Now suppose instead that $nx > n^*x^*$. Then $wnx - n^*x^* > nx - wn^*x^*$. Suppose first that $\beta t(0) > 1$ is close enough to 1 that $\Lambda(0; nx, n^*x^*, w) > 0$. Then tasks in the neighborhood of task 0 yield lower costs in West. Since $t'(i) > 0$ for all i, either there exists $J > 0$ such that $\Lambda(J; nx, n^*x^*, w) = 0$, in which case tasks with $i > J$ have lower cost in East ($\tilde{\pi}(i) < 0$) and tasks with $i < J$ have lower cost in West ($\tilde{\pi}(i) > 0$), or $(wnx - n^*x^*) > \beta t(1)(nx - wn^*x^*)$, in which case $\Lambda(i; nx, n^*x^*, w) > 0$ for all i and all tasks have lower cost in West ($\tilde{\pi}(i) > 0$ and $J = 1$). If $\beta t(0)$ is such that $\Lambda(0; nx, n^*x^*, w) < 0$, then since $t'(i) > 0$ for all i, all tasks have lower costs in East, namely, $\tilde{\pi}(i) < 0$ and $J = 0$. *Q.E.D.*

LEMMA 2: *If $w > w^*$, then $J < I$ implies $I > I^*$.*

PROOF: The proof of Lemma 1 guarantees that if $w > 1$, then $n^*x^* > nx$ implies $J = 1$. So we can limit our attention to circumstances with $nx > n^*x^*$. To establish a contradiction, we suppose that $J < I$ and $I^* > I$. Then (1) and (3) imply that $w^2 > A(nx)/A(n^*x^*)$.

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From the definition of J, we know that

$$\beta t(J) - \beta t(I) = \frac{wnx - n^*x^*}{nx - wn^*x^*} - \frac{A(nx + n^*x^*)}{wA(n^*x^*)}. \tag{15}$$

Since the denominators are both positive for $J \in (0, 1)$, the left-hand side has the same sign as

$$\begin{aligned}\Delta(n^*x^*, nx, w) \equiv{}& w^2 A(n^*x^*)nx - wA(n^*x^*)n^*x^* \\ &- A(nx + n^*x^*)nx + wA(nx + n^*x^*)n^*x^*.\end{aligned}$$

But then $w^2 > A(nx)/A(n^*x^*)$ implies that

$$\begin{aligned}\Delta(n^*x^*, nx, w) >{}& n^*x^*[A(nx + n^*x^*) - A(n^*x^*)] \\ &+ nx[A(nx) - A(nx + n^*x^*)].\end{aligned}$$

Define the the right-hand side as $\Omega(n^*x^*, nx)$ and note that $\Omega(\cdot)$ is continuously differentiable in both arguments and $\Omega(nx, nx) = 0$. Calculate the partial derivative of $\Omega(n^*x^*, nx)$ with respect to the second argument. Then $\Omega_2(0, nx) = 0$ and $\Omega_2(nx, nx) = A(nx) + nxA'(nx) - A(2nx) \geq 0$, where the inequality follows from the concavity of $A(\cdot)$. Note also that $\Omega_{12}(n^*x^*, nx) = -(nx - n^*x^*)A''(n^*x^* + nx) \geq 0$ by the concavity of $A(\cdot)$. Then, since $\Omega_2(\cdot)$ is continuous, $\Omega_2(n^*x^*, nx) \geq 0$ for all $n^*x^* \geq 0$ and $nx \geq n^*x^*$. Since $\Omega(nx, nx) = 0$ and $\Omega_2(n^*x^*, nx) \geq 0$ for all $nx \geq n^*x^*$, it follows by continuity that $\Omega(n^*x^*, nx) \geq 0$ for all $nx \geq n^*x^*$. Hence, if $w > 1$, $I^* > I$, and $nx > n^*x^*$, we obtain that $\Delta(n^*x^*, nx, w) > 0$, which implies by (15) that $J > I$. This establishes our contradiction. *Q.E.D.*

LEMMA 3: $w > 1$ *if and only if* $nx > n^*x^*$.

PROOF: We consider three mutually exhaustive cases: (i) $I \geq I^*$, (ii) $I < I^*$ and $L > L^*$, and (iii) $I < I^*$ and $L \leq L^*$.

(i) From the definitions of I and I^* in (1) and (3), $I \geq I^*$ implies

$$\frac{A(nx + n^*x^*)}{wA(n^*x^*)} \geq \beta t(I) \geq \beta t(I^*) \geq \frac{wA(nx + n^*x^*)}{A(nx)},$$

which implies that $A(nx)/A(n^*x^*) \geq w^2 > 1$. So $nx > n^*x^*$.

(ii) To establish a contradiction, suppose that $nx \leq n^*x^*$. From Figure 3(d) and (e), $I < I^*$ implies $\mathcal{E} = \emptyset$. Then

$$L = \frac{M(\mathcal{D})nx}{A(nx)} > L^* > \frac{M(\mathcal{D})n^*x^*}{A(n^*x^*)},$$

which implies $A(nx)/(nx) < A(n^*x^*)/(n^*x^*)$. But $A(\cdot)$ concave, $A(0) \geq 0$, and $nx \leq n^*x^*$ imply that $A(nx)/(nx) \geq A(n^*x^*)/(n^*x^*)$. This contradicts the supposition that $nx < n^*x^*$.

(iii) To establish a contradiction, suppose that $nx \leq n^*x^*$. Labor-market clearing implies $L = (1 - I^*)nx/A(nx)$ and

$$L^* > (1 - I^*)\frac{n^*x^*}{A(n^*x^*)} + I^*\frac{nx + n^*x^*}{A(nx + n^*x^*)},$$

since $T(I^*) > I^*$ for all I^*. From manager-market clearing, and $H = L$ and $H^* = L^*$, this implies that

$$\frac{x}{x^*} > \frac{\dfrac{1 - I^*}{A(n^*x^*)} + I^*\left(\dfrac{nx + n^*x^*}{n^*x^*}\right)\dfrac{1}{A(nx + n^*x^*)}}{\dfrac{1 - I^*}{A(nx)}}.$$

Note that $nx \leq n^*x^*$ and $w > 1$ imply that

$$\frac{c}{c^*} = \frac{\dfrac{w(1 - I^*)}{A(nx)} + \dfrac{\beta T(I^*)}{A(nx + n^*x^*)}}{\dfrac{1 - I^*}{A(n^*x^*)} + \dfrac{I^*}{A(nx + n^*x^*)}} \geq 1.$$

Equation (7) implies, since $\sigma > 1$, that $x^*/x \geq c/c^*$. Given that $T(I^*) > I^*$ and $w > 1$, then

$$\frac{x}{x^*} < \frac{\dfrac{1 - I^*}{A(n^*x^*)} + \dfrac{I^*}{A(nx + n^*x^*)}}{\dfrac{1 - I^*}{A(nx)} + \dfrac{I^*}{A(nx + n^*x^*)}}.$$

Therefore, for an equilibrium to exhibit $nx < n^*x^*$, it has to be the case that

$$\frac{\dfrac{1 - I^*}{A(n^*x^*)} + \dfrac{I^*}{A(nx + n^*x^*)}}{\dfrac{1 - I^*}{A(nx)} + \dfrac{I^*}{A(nx + n^*x^*)}} > \frac{x}{x^*} > \frac{\dfrac{1 - I^*}{A(n^*x^*)} + I^*\left(\dfrac{nx + n^*x^*}{n^*x^*}\right)\dfrac{1}{A(nx + n^*x^*)}}{\dfrac{1 - I^*}{A(nx)}}.$$

But note that $I^*/A(nx+n^*x^*)>0$ and $(nx+n^*x^*)/n^*x^*>1$, so

$$\frac{\dfrac{1-I^*}{A(n^*x^*)}+\dfrac{I^*}{A(nx+n^*x^*)}}{\dfrac{1-I^*}{A(nx)}+\dfrac{I^*}{A(nx+n^*x^*)}} < \frac{\dfrac{1-I^*}{A(n^*x^*)}+I^*\left(\dfrac{nx+n^*x^*}{n^*x^*}\right)\dfrac{1}{A(nx+n^*x^*)}}{\dfrac{1-I^*}{A(nx)}},$$

which contradicts the previous string of inequalities. *Q.E.D.*

Dept. of Economics, Princeton University, Princeton, NJ 08544-1021, U.S.A.; grossman@princeton.edu

and

Dept. of Economics, Princeton University, Princeton, NJ 08544-1021, U.S.A.; erossi@princeton.edu.

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